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The Impact of the Cosmic Microwave Background on Large-Scale Structure

Martin White

Enrico Fermi Institute, 5640 S. Ellis Ave., Chicago IL 60637

Douglas Scott

Department of Geophysics & Astronomy and Department of Physics
129-2219 Main Mall, University of British Columbia
Vancouver, B.C. V6T 1Z4 Canada

ABSTRACT

The *COBE* detection of microwave anisotropies provides the best way of fixing the amplitude of cosmological fluctuations on the largest scales. We discuss the impact of this new, precise normalization and give fitting formulae for the horizon-crossing amplitude as a function of Ω_0 and n for both open and flat cosmologies. We also discuss the relevant normalization (σ_8) at galaxy-clustering scales. Already it is clear that the inferred σ_8 can be unnaccepatably high for some of the simplest inflationary models, although many minor variants give an adequate fit. Generic topological defect models appear to fare rather badly, and it is unclear whether minor variants or improved calculations will help much. The detection and mapping of structure in the CMB anisotropy spectrum on smaller scales in the near future will enable us to achieve much stronger constraints on models.

Subject headings: cosmic microwave background — cosmology: theory — large-scale structure

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1. Introduction

The study of fluctuations in cosmology has two distinct branches, the Cosmic Microwave Background (CMB) and Large-Scale Structure (LSS). Any theory which purports to explain phenomena in one field must also be able to withstand observational scrutiny from the other. Thus any advances in the study of CMB anisotropies impact upon LSS studies.

Perhaps the most immediate impact that the CMB has made upon LSS is in the area of normalization. In order to make firm predictions, a cosmological model needs to have the amplitude of its fluctuations fixed at some specific scale. Classically, it has been standard practice to normalize models of large-scale structure at around $\simeq 10\,h^{-1}$Mpc (here the Hubble constant $H_0 = 100\,h\,\mathrm{km\,s^{-1}Mpc^{-1}}$), using a quantity related to the clustering of galaxies measured at the current epoch. The most common normalization of the 1980's was σ_8, the rms mass overdensity in spheres of size $8\,h^{-1}$Mpc. This scale was chosen, since for optical galaxies the rms there is estimated to be of order 1.

However, this approach has two basic problems. Firstly, at these scales the fluctuations are still well inside the horizon, and so their relationship to larger scale fluctuations depends on details such as their evolution since matter-radiation equality. And secondly, these scales are not sufficiently large that the fluctuations are within the linear regime. A related uncertainty is the relationship between the observed structure and the underlying mass distribution in the universe (i.e. the issue of biasing).

With the *COBE* DMR detection of CMB anisotropies (Smoot et al. 1992), it has become possible to directly normalize the potential fluctuations at near-horizon scales, circumventing the problems with the 'conventional' normalization. Thus the mass fluctuation power spectrum can now be definitively normalized, and attention is focussing (beginning with Wright et al. 1992 and Efstathiou, Bond & White 1992) on what this tells us about LSS.

In this paper we will discuss the *COBE* normalization for a wide variety of models which are currently popular, with emphasis on models of the Cold Dark Matter (CDM) type, but with general comments about models with $\Omega_0 \neq 1$, as well as non-inflationary models. We will show what impact the *COBE* data have had on our understanding of what is required of the matter power spectrum. And we will point out some of the developments which may soon come from the consideration of smaller-scale CMB experiments as the data improve.

The basic thrust is as follows:

(1) The *COBE* DMR data provides the best normalization for the largest scale fluctuations.

(2) While for any particular model it is possible to calculate the relation between the large scale normalization and the amplitude of the fluctuations on galaxy and cluster scales, in practice this involves several parameters (e.g. spectral slope and Hubble constant) whose values are not well known.

(3) In the low-Ω_0 models, the non-trivial evolution of the potential near last-scattering and between last scattering and today makes normalization of the matter power spectrum more involved.

(4) Given the *COBE* normalization, plus estimates of σ_8 (e.g. from cluster abundances), there are already tight constraints on allowed parameter ranges for any class of model.

(5) The detection of degree-scale structure in the CMB anisotropy spectrum will place quite separate constraints on combinations of cosmological parameters.

2. Power Spectra and Normalizations

A useful way of thinking about the power spectra is to view the matter and radiation curves as two separate outputs of a cosmological model, which has as inputs the cosmological parameters,

dark matter content and initial fluctuation spectrum. As Ω_0, $\Omega_{\rm B}$, h, Λ, etc. are varied, the two curves change in different ways. In addition the relative normalization between the curves is an output, so only *one* overall normalization needs to be set by comparison with data. Hence if we fix the normalization using the *COBE* anisotropy data, then we have also determined the normalization of the matter fluctuations *for any specific model*. It is important to understand that in deriving a quantity like σ_8 there are several separate effects: the precise normalization to the *COBE* data will depend on the model through the shape of the CMB power spectrum; the relative normalization of the matter power spectrum will depend on the model through different growth factors between $z \sim 10^3$ and $z \simeq 0$; and the calculation of, say, σ_8 will depend on the model through the precise shape of the matter power spectrum.

The power spectrum of CMB fluctuations is usually expressed in terms of the multipole moments C_ℓ. These are defined by expanding the two-point function of the temperature fluctuations in Legendre polynomials (assuming the model has no preferred direction):

$$\left\langle \frac{\Delta T}{T_0}(\hat{n}_1)\,\frac{\Delta T}{T_0}(\hat{n}_2)\right\rangle \equiv \frac{1}{4\pi}\sum_{\ell=0}^{\infty}(2\ell+1)C_\ell\,P_\ell(\hat{n}_1\cdot\hat{n}_2), \tag{1}$$

where $T_0 = 2.726$K is the average temperature of the CMB (Mather et al. 1994), and the angled brackets indicate an average over the ensemble of fluctuations (see White, Scott & Silk 1994).

For the matter density perturbations, the LSS data is usually expressed in terms of the power spectrum $P(k) \equiv |\delta_k|^2$, where δ_k is the Fourier transform of the fractional density perturbation

$$\delta_k \equiv \delta\left(|\vec{k}|\right) = \int d^3x\ \frac{\delta\rho}{\rho}(\vec{x})\,e^{i\vec{k}\cdot\vec{x}}. \tag{2}$$

Since standard models postulate gaussian fluctuations, specifying the power spectrum completely determines the properties of the fluctuations. As the model has no preferred direction, the spectrum depends only on the magnitude of $\vec{k}$. Another measure of $P(k)$ that is often used is the contribution to the mass variance per unit interval in $\ln k$, denoted $\Delta^2(k)$, which has the virtue of being dimensionless:

$$\Delta^2(k) \equiv \frac{d\sigma^2_{\rm mass}}{d\ln k} = \frac{k^3}{2\pi^2}P(k). \tag{3}$$

Within the context of an inflationary model, once the initial fluctuation spectrum (which we parameterize by its amplitude and spectral slope n) and the cosmological parameters are specified, both the linear theory $P(k)$ and C_ℓ can be calculated to very high accuracy (Hu et al., 1995). Hence there is no ambiguity (in linear theory) for the predictions of a specific model, although several parameters affect these predictions.

The normalization of $P(k)$ is frequently expressed in terms of

$$\sigma_8^2 \equiv \int_0^\infty \frac{dk}{k}\ \Delta^2(k)\left(\frac{3j_1(kr)}{kr}\right)^2, \quad \text{with } r = 8\,h^{-1}\text{Mpc}, \tag{4}$$

which measures the variance of fluctuations in spheres of radius $8\,h^{-1}$Mpc. Using the Press-Schechter or peak-patch methods, its value can be inferred from the abundance of clusters (Bond & Myers 1991, White, Efstathiou & Frenk 1993, Carlberg et al. 1994, Viana & Liddle 1996) to be $\sigma_8 \simeq 0.5$–0.8, with some Ω_0 dependence. Specifically Viana & Liddle (1996) find

$$\sigma_8 \simeq (0.6 \pm 0.1)\Omega_0^{-\alpha}\ , \tag{5}$$

with $\alpha \simeq 0.4$ for open CDM and $\alpha \simeq 0.45$ for ΛCDM. [More accurate fits plus a discussion of the uncertainty as a function of Ω_0 can be found in their paper]. These values are consistent with those inferred from large-scale flows (Dekel 1994, Strauss & Willick 1995) and direct observations of galaxies (e.g. Loveday et al. 1992). Note that for very low Ω_0, this implies that galaxies become anti-biased (i.e. $b \equiv 1/\sigma_8 < 1$).

Since *COBE* probes scales near the horizon size today we find it cleanest to quote the normalization inferred from *COBE* in terms of the amplitude of the mass or potential fluctuations at large scales (small k). Specifically we use $\delta_{\rm H}$, the density perturbation at horizon-crossing, which is defined through (see Liddle & Lyth 1993)

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} = \delta_{\rm H}^2 \left(\frac{k}{H}\right)^{3+n} T^2(k), \tag{6}$$

with $T(k)$ the transfer function describing the processing of the initial fluctuations. We find to very good approximation that $\delta_{\rm H}$ as determined by *COBE* is independent of both h and $\Omega_{\rm B}$, although it will depend on Ω_0 and Λ. Given $\delta_{\rm H}$, the value of σ_8 can be calculated using Eq. (4). This will introduce an additional dependence on n, Ω_0 and h.

3. The *COBE* Normalization

3.1 History

With the detection of large-angle temperature fluctuations in the CMB, the *COBE* satellite made possible (for the first time) accurate normalization of models of structure formation. Hence fitting to σ_8 became a constraint on the shape and spectral tilt of the models, rather than the primary normalization. In this context note that the *COBE* normalization does *not* predict that models are unbiased ($b \simeq 1$). Firstly the *COBE* normalization depends on the values of Ω_0 and Ω_Λ, so a single statement of this type cannot summarize the *COBE* normalization. Secondly the 'bias' depends on inferring the amplitude of the fluctuations on much smaller scales than the *COBE* data measures, and thus depends on the values of several uncertain model parameters.

The first year *COBE* data were low signal-to-noise (S/N), with the rms fluctuation having a 30% error. Fits to the full data set, the angular correlation function or the rms fluctuation all gave consistent values for the quadrupole expectation value: $\langle Q \rangle \,(n=1) = 17 \pm 5\mu$K (also known as $Q_{\rm rms-PS}(n=1)$, Smoot et al. 1992, Seljak & Bertschinger 1993, Scaramella & Vitorrio 1993, Wright et al. 1994). Note that the value of the actual quadrupole was lower, but consistent within the expected variance.

The second year of data (Bennett et al. 1994) resulted in a dramatic improvement of the S/N and a consequent increase in the degree of refinement of the analyses. These data constrain the large-scale normalization to $\sim 10\%$, with 5–7% of this being due to irremovable cosmic and sample variance. Ironically, along with the better S/N came ambiguity in the number to use for normalization (amounting to 30% discrepancy!), due in large part to a low quadrupole in the 2nd year map (Banday et al. 1994, Bunn, Scott & White 1995). Normalization of models directly to the temperature maps became essential to obtain all the information now available from the *COBE* data (Górski et al. 1994, Bond 1995, Bunn 1995). In addition highly accurate calculations of theoretical predictions and their relation to large-scale structure began to be included in the analyses (Bunn, Scott & White 1995, Bunn & Sugiyama 1995, Hu, Bunn & Sugiyama 1995, Górski, Ratra, Sugiyama & Banday 1995, Tegmark & Bunn 1995, White & Bunn 1995, Stompor, Górski & Banday 1995, Yamamoto & Bunn 1995, White & Scott 1995, Cayón et al. 1995), making the *COBE* normalization probably the most accurately known piece of information about large-scale structure.

The final installment in the *COBE* saga has now appeared. The full 4-year data have been analyzed and found to be very similar to the results of the 2-year data (Bennet et al. 1996, Bunn, private communication). The normalization is 10%, or 1σ, lower than the 2-year results, half of which comes from the change from 2-year to 4-year data and half from a "customized" cut of the galaxy based on the DIRBE data (Bennet et al. 1996). The actual quadrupole is no longer anomalously low in the 4-year data. A preliminary analysis of the 4-year data (Bunn, private communication) shows that for the range of theories we have been discussing the $\delta_{\rm H}$ values can be obtained by reducing by 10% the values fit to the 2-year data, and we have done that for all the fits quoted in §4.

3.2 Using the *COBE* Data

When normalizing to the *COBE* data one can choose to use several quantities:

(1) $\sigma(10^\circ)$, the rms temperature fluctuation averaged over a 10° FWHM beam, or some other angular scale;

(2) $\langle Q\rangle\,(n=1)$, the best-fitting amplitude for an $n=1$ Harrison-Zel'dovich spectrum, quoted at the quadrupole scale;

(3) Fits to the full sky maps, using complete theoretical calculation of the expected spectrum.

As discussed in Bunn et al. (1995) and Banday et al. (1994) there is more information in the *COBE* data than just the rms power measured, methods (1) and (2). In other words, the *COBE* data *cannot* be reduced to a single number without a significant loss of information. As mentioned previously, the choices (1) and (2) above lead to results different by 30% in the normalization $\delta_{\rm H}$. It is only with an analysis of the full sky maps for each given model that the best normalization can be obtained, and the true power of the *COBE* normalization (accurate to $\sim 10\%$) can be exploited.

3.3 From Radiation to Matter

While the amplitude of the CMB fluctuations is well determined, obtaining the normalization of the matter power spectrum from the CMB measurement can present some complications. In the simplest picture, in which large-angle CMB anisotropies come purely from potential fluctuations on the last scattering surface, the relative normalization of the CMB and matter power spectrum today is straightforward (e.g. Efstathiou 1990, White, Scott & Silk 1994): the matter power spectrum for an $\Omega_0 = n = 1$ CDM universe is

$$\begin{aligned} P(k) &= \frac{3\pi\eta_0^4}{2}\, C_2\, k\, T^2(k) \\ &\simeq 6.0\times 10^{15} C_2\,(k/h\,{\rm Mpc}^{-1})\,T^2(k) \quad (h^{-1}{\rm Mpc})^3. \end{aligned} \tag{7}$$

Here η_0 is the conformal time today: $\eta_0 \simeq 2/H_0$ if we neglect the contribution of the radiation to Ω. In models such as CDM this relation works quite well, as long as matter-radiation equality is sufficiently early (h is not too low: see Bunn et al. (1995) for further discussion).

However, in general the rise into the first peak in the CMB spectrum means that for computing the spectral shape to fit to *COBE*, Sachs-Wolfe (meaning simple potential fluctuations) is not enough; for reasonable baryon abundances the tail of the acoustic peaks is significant even at *COBE* scales. Hence the accurate C_ℓ spectrum should be fit to the data directly. For models with $\Omega_0 < 1$ the normalization is even less straightforward (see also §4). The additional effects which must be considered in this case include: the growth of perturbations from $z\sim 10^3$ until the present; the Ω_0 dependence of the potentials for fixed $P(k)$; and the effect of the decaying potentials on the propagation of photons.

Another important consideration is the possible contribution of gravity waves (tensors) to the *COBE* fluctuations. If this contribution is non-negligible then the inferred matter fluctuations are correspondingly lower. Conventionally this is defined in terms of the ratio of tensor to scalar contribution to the quadrupole: $C_2^{(T)}/C_2^{(S)}$ also wrritten as T/S. If the inflationary model is specified then this quantity is calculable, and is often related to the tilt, e.g. it is $7(1-n)$ for power-law inflation in the $\Omega_0 = 1$ and $n \simeq 1$ limit.

4. Specific Models

4.1 Critical Density Models ($\Omega_0 = 1$)

Standard CDM normalized to *COBE* has a value of σ_8 which is significantly greater then one. The over-abundance of power on small scales manifests itself in many problems, one of which is that CDM predicts too many clusters. There are several ways out of this dilemma: reducing h (to unrealistically small values); adding a component of tensors; tilting the initial power spectrum; invoking a contribution from massive neutrinos; allowing the τ neutrino to be massive and unstable; considering the possibility that $\Omega_0 \neq 1$; or abandoning the whole CDM paradigm. Most cosmologists are reluctant to pursue the last possibility, because of the conspicuous successes of this simple scheme. Reasonable changes in h, n and T/S in combination can lead to acceptable models (see White et al. 1995). A neutrino with a mass $\sim$eV or higher than usual baryon abundance would both lead to small-scale damping which could also help obtain the required shape. Any (or all) of these variants could also be considered along with the abandonment of the critical density assumption, as discussed below.

A fit to the 4-year *COBE* data for standard CDM gives

$$10^5\, \delta_{\rm H}(n) = 2.0 \exp[a(1-n)], \tag{8}$$

as a function of n with a statistical error of 7%. Here $a = 0.85$ with no gravitational waves and $a = -0.76$ with power-law inflation gravitational waves. The fit works to better than 5% for $0.8 \leq n \leq 1.2$. This normalization can be used to compute specific LSS quantities for any $\Omega_0 = 1$ model, where the transfer function is known accurately. In Fig. 2 we show some values of σ_8 vs h for a range of values of n. Specifically here we have assumed $\Omega_{\rm B} h^2 = 0.015$ from Big Bang Nucleosynthesis (BBN: see Copi, Schramm & Turner 1995, Krauss & Kernan 1995) and have allowed for either $T/S = 7(1-n)$ or $T/S = 0$. Variations in the assumed baryon fraction over the allowed BBN range have a $\sim 10\%$ effect on σ_8, but the damping can have quite significant effects at smaller scales.

4.2 Flat, Low Density Models ($\Omega_0 + \Omega_\Lambda = 1$)

If one stays with the original motivation of the inflationary paradigm, where the final state of the universe is independent of its initial state or of contrived features in the inflationary potential (and hence generically calculable), one is lead to consider only universes with vanishing spatial curvature. The desire for a low-Ω_0 can be accomodated within this picture then only if one artificially introduces a cosmological constant with $\Omega_\Lambda = 1 - \Omega_0$.

For models with $\Omega_0 < 1$ the relative normalization of the matter and radiation is not as straightforward as Eq. (7). There are several effects which come into play when normalizing the matter power spectrum to the *COBE* data in a low-Ω_0 model. The first is that, though the growth in such models is suppressed by $g(\Omega_0)$ (see Carroll et al. 1992), the potential fluctuations are proportional to Ω_0. In terms of the power spectrum, $P(k)$, we expect for fixed *COBE* normalization that $P(k) \propto (g(\Omega_0)/\Omega_0)^2$, as has been pointed out by Peebles (1984) and Efstathiou, Bond &

White (1992). For a fixed *COBE* normalization the matter fluctuations today are *larger* in a low-Ω_0 universe, and the cosmological constant model clearly has the most enhancement, since the fluctuation growth is less suppressed than in an open model.

However the growth and potential suppression are not the only effects which occur in low-Ω_0 universes. Due to the fact that the fluctuations stop growing (or in other words the potentials decay) at some epoch, there is another contribution to the large–angle CMB anisotropy measured by *COBE*. In addition to the redshift experienced while climbing out of potential wells on the last scattering surface, photons experience a cumulative energy change due to the decaying potentials as they travel to the observer. With decaying potentials, the blueshift of a photon falling into a potential well is not entirely cancelled by a redshift when it climbs out. This leads to a net energy change, which accumulates along the photon path, often called the Integrated Sachs-Wolfe (ISW) effect to distinguish it from the more commonly considered redshifting which has become known as the Sachs-Wolfe effect (both effects were considered in the paper of Sachs & Wolfe 1967). This ISW effect will operate most strongly on scales where the change of the potential is large over a wavelength. For ΛCDM models the effect is confined to the largest angles (Kofman & Starobinsky 1985), i.e. $\ell \lesssim 10$'s.

Because of this, the large-scale normalization of ΛCDM models is strongly dependent on Ω_0, with lower values of Ω_0 leading to higher normalizations. In order to obtain models with reasonable 'shape' parameters $\Gamma \simeq \Omega_0 h \simeq 0.25$, and which are not anti-biased on galaxy and cluster scales the models need a spectral tilt with $n < 1$ (e.g. Scott, Silk & White 1995, Klypin, Primack & Holtzman 1995). For the models with spectral tilt the *COBE* normalization can also be reduced by introducing a component of gravity waves.

For Λ models with tensors there is a correction to the well known relation (Davis et al. 1992) $C_2^{(T)}/C_2^{(S)} = 7(1-n)$, which reduces the tensor component (Knox 1995) at fixed n. This arises because the predicted scalar quadrupole increases more than the tensor quadrupole as Λ is increased (i.e. the effect is tied to the fact that the ratio is defined at $\ell = 2$ where the ISW contribution to the scalar C_ℓ is large). This was originally neglected in White & Bunn (1995), but has been included (in addition to the n dependence of the correction) in all the results of this paper (see also Turner & White 1995).

A fit to the 4-year *COBE* data for flat models gives the horizon-crossing amplitude

$$10^5\,\delta_{\rm H}(n,\Omega_0) = 2.0\;\Omega_0^{-0.775-0.04\ln\Omega_0}\exp\left[a(1-n)\right]\,, \qquad (9)$$

where $a = 0.85$ with no gravitational waves and $a = -0.76$ with power-law inflation gravitational waves. This fit works to better than 5% for $0.1 < \Omega_0 \leq 1$ and $0.8 \leq n \leq 1.2$, and again the statistical uncertainty is 7%.

4.3 Models with Spatial Curvature ($\Omega_0 \neq 1, \Lambda = 0$)

Perhaps more natural from the point of view of fine tuning, models which are open also have a weaker Ω_0 dependence in their *COBE* normalization than flat Λ-models. For the open models however the epoch of last scattering and the transitions from radiation to matter to curvature domination are not well separated in scale (see Fig. 3). Thus the ISW effect dominates the spectrum for angular scales larger than $\sim 1^\circ$. This makes the relation between the CMB anisotropy and the large scale matter power spectrum difficult to guess without a detailed calculation, even when the effects of curvature are neglected! The dependence of $\delta_{\rm H}$ on Ω_0 is contrasted with the simple scaling of $g(\Omega_0)/\Omega_0$ in Fig. 1.

Specific calculations of inflation with $\Omega_0 < 1$ now exist (e.g. Lyth & Stewart 1990, Ratra & Peebles 1994, Bucher, Goldhaber & Turok 1995). These models give robust predictions for the

power spectrum around the curvature scale, which should now be preferred over simple power-law assumptions (Kamionkowski & Spergel 1994). There is one additional complication in open models, the existence of modes with wavelength larger than the curvature scale. Fortunately these super-curvature modes in the inflationary theories do not change the matter normalization for reasonable Ω_0, although they do affect the *COBE* goodness-of-fit (Yamamoto & Bunn 1995). These modes can also be suppressed by suitable tuning of the inflationary potential.

For the open models the relative normalization of the scalar and tensor modes predicted by inflation is currently an unresolved issue. For these models there is a 'feature' in the power spectrum near the scales relevant for structure formation, hence non-negligible tensor mode contribution and departures from scale invariance (and power law spectra) are perhaps more likely than in the flat inflationary models with featureless spectra. A definitive statement awaits more theoretical work in this area.

A fit to the 4-year *COBE* data for open models gives the horizon-crossing amplitude

$$10^5\,\delta_{\rm H}(n,\Omega_0) = 1.89 + 1.98(1-n) + 1.95\Omega_0 - 1.87\Omega_0^2, \tag{10}$$

for the no gravitational wave case only. This fit works to better than 5% for $0.2 < \Omega_0 \leq 1$ and $0.9 \leq n \leq 1.1$, and again the statistical uncertainty is 7%.

4.4 Baryonic Isocurvature Models

Although the CDM-like inflationary-inspired models have been very successful, it is still possible that this success is misleading, and that a whole different paradigm might fit everything better. One contender for an 'on the other hand' class of models are those with only baryons as dark matter, with ad hoc power law initial conditions in the isocurvature rather than adiabatic mode (Peebles 1987). These Primordial Isocurvature Baryon (PIB) models can also be compared directly with data. However there are many tunable parameters, so it is difficult to make unambiguous predictions.

Generically such models give an effective slope on *COBE* scales which is rather high. No open PIB model fits the data, unless the initial conditions are contrived (Hu, Bunn & Sugiyama 1995). Some flat Λ-dominated models survive the stringent observational constraints, although it would be fair to say that these models have to try hard to fit. A full discussion of the effective $\delta_{\rm H}$ and σ_8 fits for PIB is beyond the scope of this comment.

Detailed normalization to the *COBE* data requires a clear idea of the super-horizon fluctuations, which is lacking here. Direct comparison of CMB and LSS data on $\sim 100\,h^{-1}$Mpc scales may provide the most conclusive test: the relationship between ΔT and the underlying potential Φ is fundamentally different in the isocurvature case.

4.5 Defect Models

An alternative to inflation is the idea that fluctuations may be generated by the dynamics of cosmic defects. The two most well known examples are cosmic strings and textures. Due to the non-linearity inherent in the evolution of these defects, it has been difficult to perform accurate calculations of the predictions of these theories, making them something of a 'moving target' for experimentalists. However the *COBE* detection of fluctuations and the CMB may hold the key to ruling out or confirming these theories once and for all. Let us deal first with the question of the impact of *COBE* on these theories.

In defect models the fluctuations in the matter and radiation are generated not in the very early universe, but rather by the motions of the defects as the universe evolves. This means that the relation between the temperature fluctuations and the gravitational potentials is more similar to

isocurvature models than adiabatic models. The extra wrinkle is that the evolution of the defects is not coherent over the age of the universe, so rather than obtaining $\Delta T \simeq 2\Phi$ as in the isocurvature case, this is reduced (by a factor akin to the $\sqrt{N}$ appearing in a random walk) to around $\frac{4}{3}\Phi$ (Pen, Spergel & Turok 1994) or $\sqrt{2}\Phi$ (Stebbins 1992, Jaffe, Stebbins & Frieman 1994).

However, this means that for fixed ΔT (from *COBE*) the predicted potential or matter fluctuations at large scales are much less than for inflationary models. The exact result depends on the modelling of the evolution of the defects, but a lower normalization along with a slightly steeper than scale-invariant spectrum seem to be fairly generic. A preliminary calculation for the case of texture models gives $\delta_{\rm H}/\sqrt{C_{10}}$ nearly an order of magnitude lower than standard CDM (see Fig. 1)! With the new abundance of data from galaxy surveys and velocity flows on large scales, such a low normalization is a serious problem for these models (see also Perivolaropoulos & Vachaspati 1994). At scales $\sim 100h^{-1}$Mpc the type of dark matter and the unknown cosmological parameters (e.g. h) which affect the shape of the power spectrum do not lead to large uncertainties in the predictions. Also the degree of non-gaussianity is less than at smaller scales and the fluctuations in the matter are well in the linear regime. These uncertainties have been a large part of the difficulty in ruling out such models in the past.

String models with CDM making up the dark matter are not a good fit to the galaxy data, irrespective of the normalization. So the most promising string models are Hot Dark Matter dominated, e.g. by massive neutrinos. Although the situation seems to be better for these string models than for textures, it still seems that there is substantially less power in the matter fluctuations than for inflationary models (Hindmarsh & Kibble 1995, Albrect & Stebbins 1992, Coulson et al. 1994, Allen et al. 1994) as can be seen in Fig. 4. The important point is that strings normalized to *COBE* have to generate adequate potentials on scales of 10–100 h^{-1}Mpc to explain the LSS data, and observed velocity flows. Irrespective of the uncertainties introduced by our ignorance of how to make galaxies in this picture, there ought to be robust predictions for the LSS data.

5. Degree-Scale CMB Data

The present status of degree-scale CMB data is rather uncertain, although few people doubt that genuine fluctuations are now being routinely detected (see e.g. Scott, Silk & White 1995, Bond 1996). However, the future looks very bright for this field, and we should be learning a lot from such fluctuations within a few years. One point we have emphasized here is that a clean test between classes of model is through the relative matter to radiation (i.e. Φ to ΔT) normalization, where both can be measured at *the same scale*.

As well as this, a combination of the shapes of the CMB and LSS power spectra will provide a detailed set of constraints on cosmological parameters and models of structure formation. There are different dependencies on Ω_0, Λ, h, Ω_B, T/S, reionization history, etc., which ought to allow us to pin down the values of these currently unknown quantities (Scott & White 1994).

Detailed extraction of the parameters awaits the powerful data-set obtainable from a future satellite mission (Scott & White 1995, Jungman et al. 1996). Such a prospect is certainly on the horizon. But even with ground- and balloon-based data, it should be possible to see general features in the degree-scale power spectrum. Certainly we expect to be able to discriminate between inflationary-inspired models and PIB-type models from basic shape considerations. The proposed long duration balloon experiments in concert with interferometers will no doubt give a model dependent estimate of Ω_0 within the next 5 years.

For topological defects, the small angular scale microwave background will be a very important discriminant. In earlier defect work high redshift reionization was assumed, both because defects are likely to seed structures at early times and because of the technical simplifications involved. However early reionization does not *have* to occur in defect models, and in its absence significant

degree scale CMB anisotropy is predicted. Recent work shows that for texture models there are peaks in the CMB spectrum (Crittenden & Turok 1995, Durrer et al. 1995) though there is some doubt as to their size. The peaks have the general character of an isocurvature spectrum because the potentials are generated around horizon crossing by the evolution of the defects (Hu & White 1996). For the cosmic string models, the decoherence of the source is likely to cause the peaks to 'merge' into one bump (Albrecht et al. 1995) situated at $\ell \sim 500$ in flat models (at smaller scales in open string models), which should be easily distinguishable from the coherent scenarios with upcoming CMB measurements (Albrecht & Wandelt 1996).

6. *COBE* and the Cosmological Constant

It is interesting to compare the open and ΛCDM models in the light of the *COBE* normalization. As is well known, LSS does not strongly differentiate the two low-Ω_0 variants. However at low Ω_0 the *COBE* normalization for the two is very different, so that in the open case anti-bias is not necessary. However, other observations (e.g. object abundances) shift as well, so there is still a substantial region of parameter space where both models fit the available data (with the possible exception of newer large scale velocity data, Kolatt & Dekel 1996). In fact the allowed regions of Ω_0, h and n tend to be very similar in the two models, with the open models preferring a slightly higher value of n (neither theory is necessarily in conflict with the degree scale data due to the possibility of spectral tilt and early reionization, in contrast to claims in Ostriker & Steinhardt 1995 and Ganga et al. 1996).

For the range of Ω_0 now favoured there is only a small gain in age from choosing a ΛCDM rather than open CDM model. So the principle motivation for introducing the cosmological constant is in maintaining consistency with standard inflationary models rather than because of an age crisis. A higher degree of anti-bais is generic to the Λ models, which could prove to be a strong observational discriminant between the two types of theories.

7. Conclusions

The measurement of primordial CMB fluctuations, particularly with the *COBE* DMR experiment, allows for a precise normalization of cosmological theories. However, it is important to keep in mind that any statement about the strength of clustering on large scales is strongly dependent on the details of the theory. A useful way to present this normalization is in terms of a quantity like the perturbation amplitude at horizon-crossing, $\delta_{\rm H}$; given the *COBE* data this quantity is largely determined by the values of Ω_0, Λ and n, for inflationary models, and more generally by the exact relationship between Φ and ΔT.

Fig. 1 shows the ratio of $\delta_{\rm H}/\sqrt{C_{10}}$. We show how this ratio depends rather differently on Ω_0 for the two cases of open or flat backgrounds. We also show that the simple scaling of growth rate divided by Ω_0 is not such a good approximation, which is because the potentials are not constant at late times in these models. This is indicated in Fig. 3, where it can be seen that for an open universe there is essentially no time when both radiation and curvature can be neglected, and hence the potential is almost always evolving to some extent.

For defect models the relationship between the temperature fluctuation ΔT and the potential Φ (which is ultimately related to the strength of the LSS) is very different. Indeed the approximate calculations suggest that for the same ΔT, these models tend to have about 4 times lower Φ (Fig. 1), and hence 16 times lower power for the same *COBE* normalization, as indicated in Fig. 4. One question for future study is how robust this normalization ratio is to modelling of the defects, and perhaps more importantly from the point of view of constructing viable models, what Ω_0 scaling this ratio has. It is clear that at present different calculations can give somewhat different results for this number. It is also clear that it depends on rather precise details of the type of defect theory,

and so specific models may be developed which fit the data much better. It is nevertheless true that at the level of getting $\delta_{\rm H}/\sqrt{C_{10}}$ right, the CDM-like theories work rather well, while today's calculations of generic defect models do not.

For a well-defined theoretical model, with all parameters specified, it is straightforward to obtain the relevant LSS numbers. Quantities, such as σ_8, can be calculated from the best-fitting value of δ_H, together with the accurate transfer function for each specific model. It is also necessary to consider whether gravitational waves exist in your theory, since they affect the large-angle CMB anisotropies, and hence the relative normalization to the scalar matter fluctuations. It is just as important to decide what slope to use for your power spectrum initial conditions, since realistic early universe theories may lead to $n \neq 1$. Since there are a range of possible parameter variations, we only calculate σ_8 here for the simple example of $\Omega_0 = 1$, shown in Fig. 2, as a function of h for a range of n. For other models σ_8 (or indeed any similar quantity) can be calculated using equations (4), (6), (8), (9) and (10).

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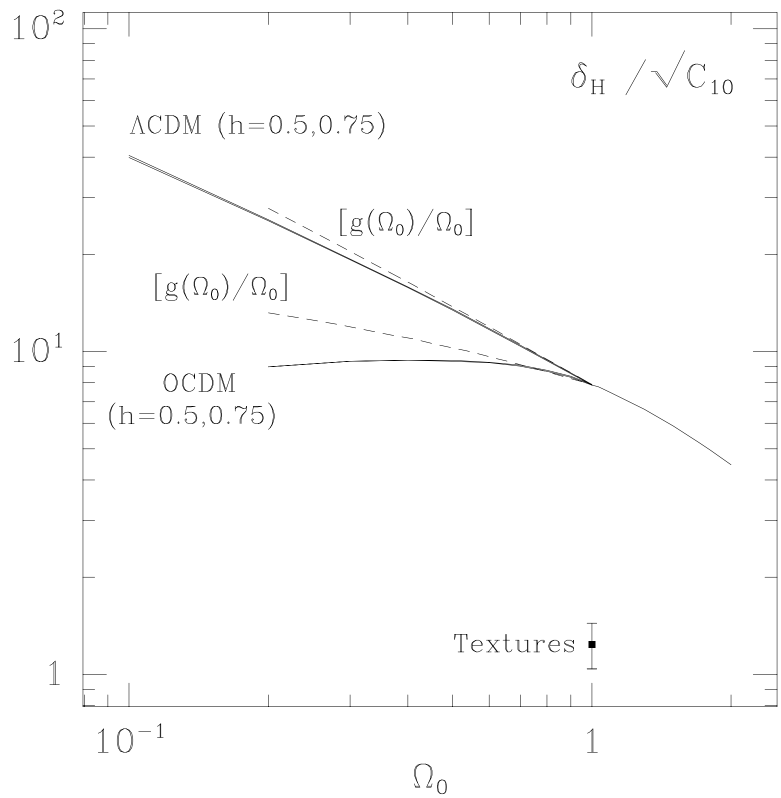

Figure 1: The relative normalization of the matter and radiation power spectra for open, flat and closed CDM models as a function of Ω_0. For any set of cosmological parameters the (dimensionless) ratio of the large-scale matter power spectrum $\delta_{\rm H}^2$ to the large-angle CMB spectrum C_{10} is fixed. Notice that the relation is almost independent of the Hubble constant h. Also shown (dashed) are the results assuming that only potential fluctuations on the last-scattering surface contribute to the CMB anisotropies on *COBE* scales, in which case the ratio depends on the growth of perturbations between last-scattering and today, $g(\Omega_0)$, and the size of the potential, Ω_0, as shown on the figure. The failure of this approximation for large Ω_Λ and almost all open/closed models is due to extra anisotropies generated by the evolution of the potential between last-scattering and today. The solid square is the predicted amplitude on large-scales for a Texture model, taken from Pen, Spergel & Turok (1994). The transfer function for Textures has more small scale power than for CDM, but the very low normalization of this theory causes problems in fitting large-scale velocities.

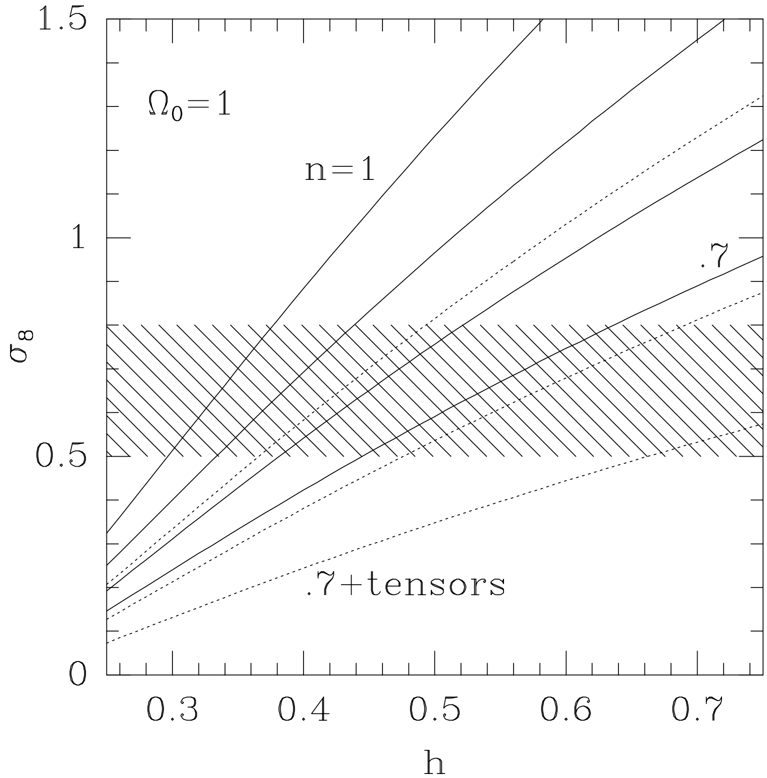

Figure 2: The mass variance in spheres of radius $8\,h^{-1}$Mpc, σ_8, for critical density CDM models with spectral tilt $n = 0.7, 0.8, 0.9, 1.0,$ as a function of Hubble constant h. The solid lines assume that only the scalar fluctuations contribute to *COBE* while the dashed lines assume that tensors contribute in the ratio $T/S = 7(1-n)$. The hatched area shows a conservative range of σ_8 values from the abundance of clusters.

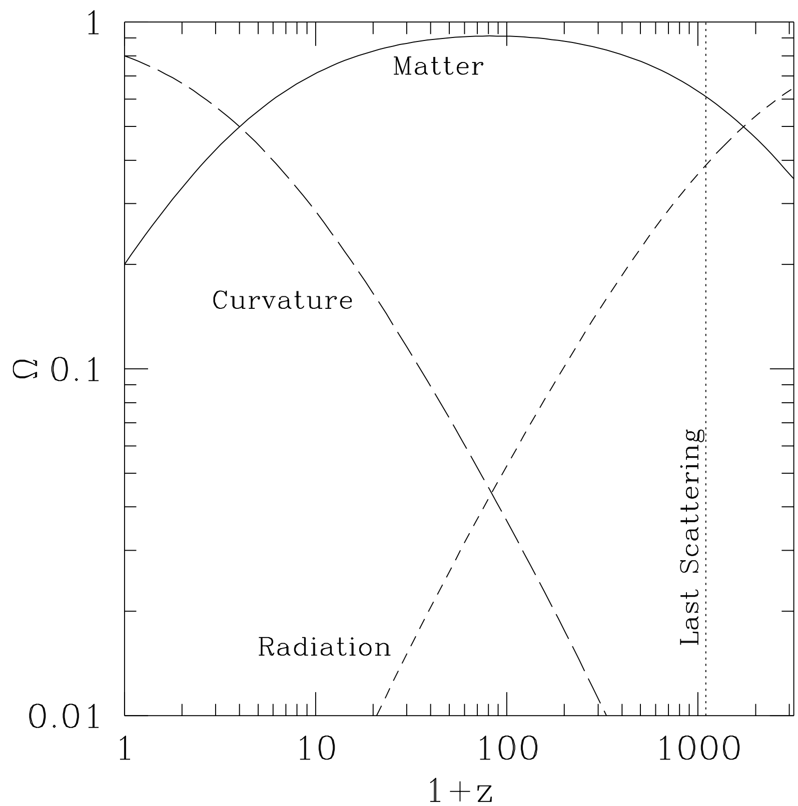

Figure 3: The contribution to the total density of matter Ω from matter, radiation and curvature in a model with $\Omega_{\rm mat} = 0.2$ and $h = 0.6$. Notice that last-scattering (the vertical dashed line) and matter-radiation equality are very close together, indicating that the usual approximation of matter-domination at $z \sim 10^3$ fails for an open universe. Also note that there is only a very small range of redshift where $\Omega_{\rm mat}$ dominates the expansion (e.g. where $\Omega_X < 0.1\Omega_{\rm mat}$, with X being curvature or radiation). Hence the gravitational potentials will almost always be evolving.

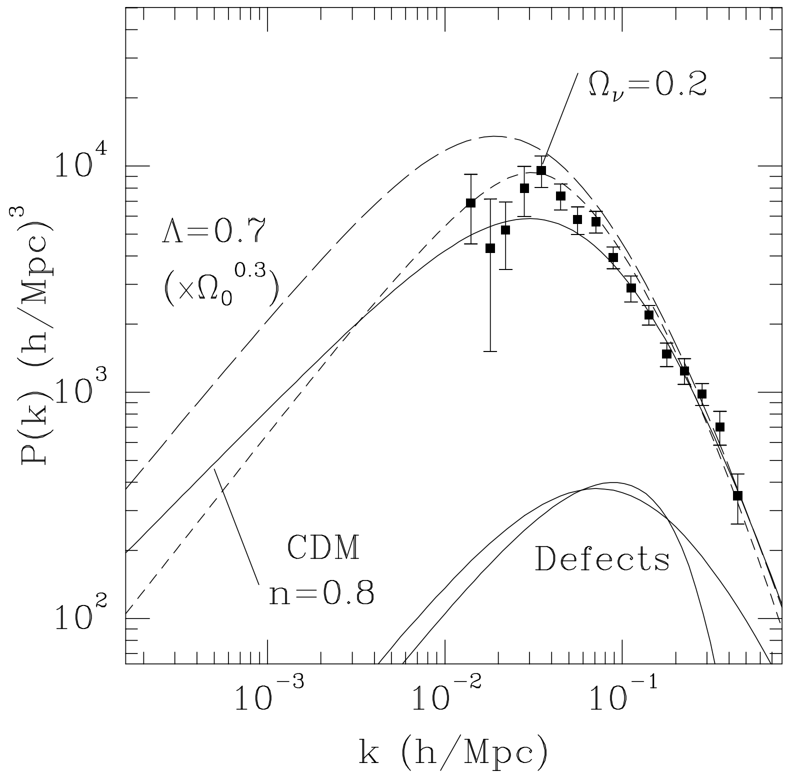

Figure 4: The matter power spectrum $P(k)$ for 3 CDM variants and two defect models. The upper solid line is a tilted CDM model with $n = 0.8$, the long-dashed line is a ΛCDM model with $\Omega_0 = 0.3$ and $h = 0.8$ and the short-dashed line is a model with $\Omega_\nu = 0.2$ in massive neutrinos. The solid lines at lower right are models based on global textures and on strings+HDM (the model with less short scale power). All models have been normalized to the 4-year *COBE* data. The data points are from the compilation of Peacock & Dodds (1994). The ΛCDM model has been multiplied by $\Omega_0^{0.3}$ as described in Peacock & Dodds (1994).